Exam. Code : 211001
Subject Code : 4956

M.Sc. Mathematics 1st Semester (Batch 2021-23)

DIFFERENTIAL EQUATIONS

Paper—MATH-555

Time Allowed—3 Hours] [Maximum Marks—100

Note :—Attempt **FIVE** questions in all, selecting at least **ONE** question from each section. The **fifth** question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. (i) Show that t, t^2 and t^3 is a fundamental set of the differential equation :

$$t^3 \frac{d^3}{dt^3} - 3t^2 \frac{d^2x}{dt^2} + 6t\frac{dx}{dt} - 6x = 0 \text{ on } [a, b].$$ 10

(ii) Define self adjoint equation with an example. Also find the necessary and sufficient condition for a second order homogenous linear differential equation to be self adjoint. 10

2. (i) Find the real eigenvalues and the corresponding eigenfunctions of the boundary value problem :

$$y'' - \lambda y = 0,\ y(0) + y'(0) = 0,\ y(1) = 0$$ 10

(ii) Discuss the existence and uniqueness of solution of the initial value problem :

$$\frac{dy}{dx} = y^{\frac{2}{3}},\ y(x_0) = 0$$ 10

SECTION—B

3. (i) Use the Laplace transform to solve the initial value problem :

$$\frac{dy}{dt} - y = e^{3t}, y(0) = 2 \qquad 5$$

(ii) State and prove Convolution theorem for Laplace transform and apply it to find $L^{-1}[H(s)]$, where

$$H(s) = \frac{1}{s(s^2+1)}. \qquad 15$$

4. (i) Use Laplace transform to find the solution of the system

$$\frac{dx}{dt} - 6x + 3y = 8e^t$$

$$\frac{dy}{dt} - 2x - y = 4e^t \text{ such that } x(0) = -1, y(0) = 0.$$

15

(ii) State and prove linearity property of Laplace transform. 5

SECTION—C

5. (i) (a) State and prove the linear property of Fourier transform. 5

(b) If Fourier transform $F\{f(x)\} = F[s]$, prove that $F\{f(x)e^{iax}\} = F[s+a]$. 5

(ii) Express the function $f(x) = \begin{cases} 1, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$ as a Fourier integral. 10

6. (i) Solve the differential equation

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = \cos t$$

using Fourier transform. 10

(ii) Find the Fourier transform of

$$f(x) = \begin{cases} 1-|x|, & \text{if } |x| < 1 \\ 0, & \text{if } |x| > 1 \end{cases} \qquad 10$$

SECTION—D

7. (i) Prove that $\int_{-1}^{1} x P_{n-1}(x) P_n(x)\,dx = \frac{2n}{4n^2-1}$. 10

(ii) Explain orthogonal property for Hermite polynomials. 10

8. (i) Prove the recurrence relation :

$$\frac{d}{dx}(J_n(x)) = \frac{-n}{x}J_n(x) + J_{n-1}(x) \qquad 10$$

(ii) State and prove Rodrigue's formula for Leguerre polynomials. 10